C23 Forensic Elasticity in the Air, on Land, and Sea

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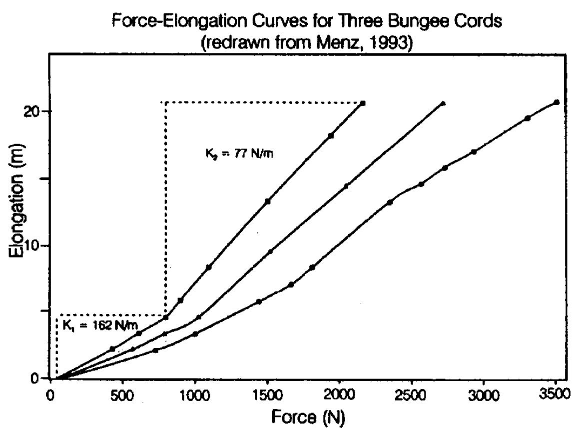
The goals of this presentation are to provide forensic engineers and scientists with an increased appreciation of the role of elasticity in the dynamics and analysis of industrial and recreational accidents.

Introduction: The analysis of bungee jumps, scaffold drops, and "rocket rope" water-tubing mishaps involves Hooke's Law, harmonic motion, and Young's Modulus. The manner in which these elements enter into understanding untoward events involving elasticity is illustrated by two accident investigations carried out by our office: (1) that of a fatal fifty-foot drop of a painting platform supported by steel cables; (2) that of a severely incapacitating and disfiguring injury to a young woman being towed across a body of water on an inner tube that was connected to a motor boat by a bungee-cord-like tow rope.

The Physics: In describing the distortion (strain) of materials under stress, a distinction is made between "elastic" strain and "plastic" strain. The former is strain that goes away completely once the stress that caused it is removed. The latter is strain that does *not* go away completely even after the stress that caused it is removed. As long as the stress applied to a material does not exceed its — reasonably enough named — "elastic limit" for that material, the resulting strain is elastic. For most materials, the first part of the elastic range is linear. For one-dimension stress, this range is described by Hooke's Law, which in extrinsic form is usually written $F = -kx$, where "x" is the change in length imposed on the object and F is the "restoring force," the force that resists the imposition of the force. Usually, "k" is called the "force constant," and obviously has dimensions of force/distance, for example, lbs/foot in the U.S.-Burmese system of units and N/m in the Système Internationale. One can also express this relationship referring to the intrinsic properties of the material, using the Young's Modulus "Y"[2] characterizing the material. Hooke's Law (the "spring equation") in terms of the Young's Modulus for the material in question becomes: $F/A = Y(\Delta L/L)$, where ΔL is the change in length of an item that had an unstretched length of L, and A is the crosssectional area of the item. Figure 1 depicts the elongation/force characteristics for three different commercial bungee cords made for jumping and is replotted from data published by Menz[2]; Figure 2 reflects our measurements on a commercial tow rope that incorporated an elastic section, to be discussed further in the final section. Note that in every case, the elastic characteristics were bi-modal, with the cord not going from an elastic region to a plastic region, but rather from an elastic region characterized by one force constant to a second elastic region characterized by a second, lower force constant. During oscillation, the motion of an object at the end



of any of these cords is more complicated than simple harmonic motion.

Figure 1

The Wire Cable Problem: This case involved one serious injury and one fatal injury as two men fell 50 feet along with the platform they had been standing on when a strong wind came up as they were painting the side of a large gas-storage tank in Lynn, Massachusetts. The scaffold had been suspended from the top of the tank by two quarter-inch braided steel cables. It was established that these cables had been improperly rigged, which lead to suit being filed against the company responsible for the rigging. That company's primary defense was to assert that the forces associated with the wind were so great that the cables would have failed regardless of how they were rigged. In particular, they relied on a report that the platform had risen a foot in the air in the wind, then had dropped vertically, and that this was the origin of the failure. In support of this theory, the defense expert put forth an original theory about how the jolt when the platform fell one foot while supported by the cables would have set up an energy wave that "blew out" one of the cables, causing the disaster. The

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authors approached the problem classically, examining the force that the weight of the platform and its contents would have exerted on the steel cables after they had dropped a distance of one foot under the force of gravity. The problem was very similar to that of determining the force with which an object hits the ground, where one knows the momentum change, but has to calculate the time over which the change occurred.[4] In the case of the cables, the problem reduced to the force constant K of the cables, since one knows the kinetic energy that must be converted into potential energy of the stretched cables.[5] This in turn gives us the "maximum stretch" and hence the maximum force on the cables, $K\Delta L$, providing that the elastic limit was not exceeded. In the event, the maximum force that would have occurred under the conditions hypothesized by the defendant was shown to be considerably below the yield strength of the cables. Following a jury trial in Boston, a verdict in favor of the plaintiffs was returned and damages assessed against the company that attached the cables to the tank. The calculations that apparently convinced the jury involve straightforward Hooke's Law application and will be illustrated.

The Rocket Rope Problem: Approximately 30,000 "rocket ropes" were sold for use in water sports, particularly for towing a rider on an inner tube at high speed behind a motor boat. Out of the 30,000 such ropes that were sold, two purchased at a particular vendor in Southern New Hampshire resulted in injury to their users. According to the manufacturer, which has ceased its sales of the item, there were no other injuries anywhere with the product. Indeed, it does take special circumstances to cause the rope to behave in a dangerous way, or, rather a particular constellation of circumstances, including in particular the weight of the rider and the speed of the boat. With the "right" combination, the ride contains several distinct phases. In the first phase, the rider and tube sit low in the water while the motor boat speeds away, stretching the rope as it goes. When the stretch has reached the point where the restoring force is high enough to move rider and tube out of their "hole," the tube rises to the water's surface. At this point, the force required to keep the tube moving can (depending on the speed of the boat) suddenly become much less than the force being applied to it by the (stretched) rope. The tube and rider therefore rapidly accelerate; as they do they begin to overtake the boat. Depending on the particular combination of fixed and variable constraints, the speed of the rider and tube may get to be so high that they continue to overtake the boat even after the force from the rope has fallen to zero. This phase lasts only a very short time, the ability of a non-streamlined object to "coast" across the water being limited. Nevertheless, it is possible during that time for the rope to become slack, and if it does, it is possible for it to wrap around the rider during the brief interval before the speeding boat pulls it taut again for the next rocketing forward. Alternately, if the operator of the boat stops his boat momentarily after the phase in which the rider has overtaken the boat, there will be ample opportunity for the unsuspecting rider to become ensnared in the rope before giving the boat operator the signal to speed up again, with disastrous results.

To first order, the dynamics of the tuber being pulled by the "rocket rope" can be modeled by a mass being pulled across a sticky surface by an elastic tether, the far end of which is moving at a constant speed, where the velocity of the mass is perturbed by its oscillation at the end of the tether and where there is a threshold speed for the mass below which the mass sticks to the surface, and remains stuck until the increasing restoring force of the tether reaches a sufficient magnitude to pull it away. Alternatively viewed, this is a problem that can be treated by considering the there to be a huge difference between the static and sliding coefficients of friction between mass and surface.

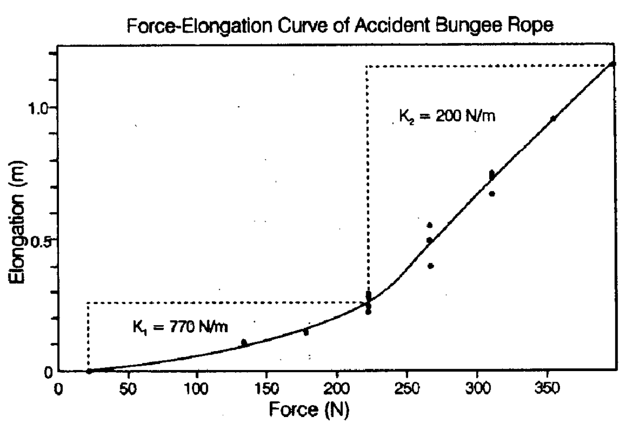

Figure 2

1 With apologies to the U.S. Marine Corps.

2 With further apologies to the engineers who tend to designate Young's Modulus by "E," a symbol

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physicists like to reserve for other things.
3 Menz, P.G. The physics of bungee jumping. *The Physics Teacher,* 31 Nov 1993, 483-7.
4 See, for example, Comments on why the question *"How Hard did it Hit?" is usually unanswerable without first answering the question "How Long did it Hit?" and a Couple of Experimental Suggestions for Working around Operational Fracture-Force Standards,* Thomas L. Bohan, delivered February, 1992, at the 44th Annual Meeting of AAFS, New Orleans, LA.
5 One of the authors (TLB) gratefully acknowledges a suggestion from John M. Orlowski, PE, CSP, which led to the quick solution of this problem.

Elasticity, Bungee Cords, Cables

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